Problems and Solutions Manual

*to accompany*

Derivatives: Principles & Practice

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Chapter 1. Futures & Options - Overview

1. **Question**: What is a derivative security?

   **Answer**: A derivative security is a financial security whose value depends on (or derives from) other, more fundamental, underlying variables such as the price of a stock, a commodity price, an interest rate, an exchange rate, or an index level. The underlying may also be a derivative security itself – there are many derivatives that are written on other derivatives.

2. **Question**: Give an example of a security that is not a derivative.

   **Answer**: An interest rate is not a derivative. It is a fundamental economic quantity reflecting the value of money.

   A stock is also typically viewed as a “primitive” (rather than a derivative) security. However, a stock may also be viewed as a derivative: it represents a claim on the cash flows generated by the assets of the issuing firm. Indeed, viewing equity as a derivative written on the underlying firm’s asset value is fundamental to the “structural model” approach to studying corporate credit risk. We examine structural models in Chapter 32.

3. **Question**: Can a derivative security be the underlying for another derivative security? Give an example.

   **Answer**: Yes, it can. The simplest example is an option (say, a call) that gives you the right to purchase another option (say, a put written on some underlying stock). In this case, the first option is called a “compound” option; it is an option on an option, in this case, a call on a put. Compound options are studied in Chapter 18. Compound options also play a role in the structural models studied in Chapter 32.

4. **Question**: Derivatives may be used for both hedging and insurance. What is the difference in these two motives?

   **Answer**: The objective of hedging, whether with a derivative or otherwise, is to eliminate the risk associated with an existing market commitment and to create a net position that is “risk-free.” That is, the hedge nullifies existing risk; in so doing, it eliminates both upside and downside potential from market moves. Futures and forwards are instruments for hedging risk.

   “Insurance” is one-sided protection. It guards against unfavorable events (“downside risk”) even while allowing full upside potential to be realized. Options are instruments that provide financial insurance.

5. **Question**: Define a forward contract. Explain at what time are cash flows generated for this contract. How is settlement determined?

   **Answer**: A forward contract is an agreement to buy or sell an asset at a future date (denoted \( T \)), at a specified price called the delivery price (denoted \( F \)). Denote the initial date (the inception date or the date of the agreement) by \( t = 0 \). At inception there are no cash flows on a forward contract. At maturity, if the then-prevailing spot price \( S_T \) of the underlying asset is greater than \( F \), then the buyer (the “long position”) has gained \( S_T - F \) via the forward while the seller (the “short position”) has correspondingly lost \( S_T - F \). Depending on contract specifications, the settlement may either be in cash (the seller pays the buyer \( S_T - F \)) or physical (the seller delivers the asset and receives \( F \)). If \( S_T < F \), the buyer loses \( F - S_T \) and the seller gains this quantity.
6. **Question**: Explain who bears default risk in a forward contract.

**Answer**: Default arises if, at maturity, one of the parties fails to fulfill their obligations under the contract. Default risk only matters for the party that is "in the money" at maturity, that is, that stands to profit at the locked-in price in the contract. (If the spot price at maturity is such that a party would lose from performing on the obligation in the contract, counterparty default is not a problem.) Prior to maturity, since either party may finish in-the-money, both parties are exposed to default risk.

7. **Question**: What risks are being managed by trading derivatives on exchanges?

**Answer**: An important one is counterparty default risk. In a typical futures exchange, the exchange interposes itself between buyer and seller and guarantees performance on the contract. This reduces significantly the default risk exposure of both parties. Further, daily settling of marked-to-market gains and losses ensures that the loss to the exchange from an investor’s default is limited to at most one day’s settlement amount (and because of maintenance margins is usually less than even this; see Chapter 2 for a description of the margining process).

8. **Question**: Explain the difference between a forward contract and an option.

**Answer**: A forward contract is an agreement to buy or sell an asset at a future date (denoted $T$), at a specified delivery price (denoted $F$). The agreement is made at time $t = 0$ for settlement at maturity $T$. An option is the right but not the obligation to buy (a “call” option) or sell (a “put” option) an asset at a specified strike price on or before a specified maturity date $T$. In comparing a long forward contract to a call option, the main difference lies in the fact that the forward buyer has to buy the stock at the forward price at maturity, whereas in a call option, the buyer is not required to carry out the purchase if it is not in his interest to do so. The forward contract confers the obligation to buy, whereas the option contract provides this right with no attendant obligation.

9. **Question**: What is the difference between value and payoff in the context of derivative securities.

**Answer**: The value of a derivative is its current fair price or its worth. The payoff (or payoffs) refers to the cash flows generated by the derivative at various times during its life. For example, the value of a forward contract at inception is zero: neither party pays anything to enter into the contract. But the payoffs from the contract at maturity to either party could be positive, negative, or zero depending on where the spot price of the asset is at that point relative to the locked-in delivery price.

10. **Question**: What is a short position in a forward contract? Draw the payoff diagram for a short position at a forward price of $103, if the possible range of the underlying stock price is $50-150.

**Answer**: A short position in a forward is where you are the seller of the forward contract. In this case, you gain when the price of the underlying asset at maturity is below the locked-in delivery price. The payoff diagram for this contract is as shown in the following picture. When the price of the stock at maturity is the delivery price of $103, there are neither gains nor losses.
11. **Question**: Forward prices may be derived using the notion of absence of arbitrage, and market efficiency is not necessary. What is the difference between these two concepts?

**Answer**: Absence of arbitrage means that a trading strategy cannot be found that creates cash inflows without any cash outflows, i.e., creates something out of nothing. Efficiency, as that term is used by financial economists, implies more: not only the absence of arbitrage but that asset prices reflect all relevant information.

12. **Question**: Suppose you are holding a stock position, and wish to hedge it. What forward contract would you use, a long or a short? What option contract might you use? Compare the forward versus the option on the following three criteria: (a) uncertainty of hedged position cash-flow, (b) Up-front cash-flow and (c) maturity-time regret.

**Answer**: If a forward contract is to be used, then a short forward is required. Alternatively, a put option may also be used. The following describes the three criteria for the choice of the forward versus the option.

- Cash-flow uncertainty is lower for the futures contract.
- The futures contract has no up-front cash-flow, whereas the option contract has an initial premium to be paid.
- There is no maturity-time regret with the option, because if the outcome is undesirable, the option need not be exercised. With the futures contract there is a possible downside.

13. **Question**: What derivatives strategy might you implement if you expected a bullish trend in stock prices? Would your strategy be different if you also forecast that the volatility of stock prices will drop?

**Answer**: If you expect prices to rise, there are several different strategies you could follow: you could go long a forward and lock in a price today for the future purchase; you could buy a call which gives you the right to buy the stock at a fixed strike price; or you could sell a put today, receive a premium, and keep the premium as your profit if prices trend upward as you expect.
The volatility issue is a bit trickier. As we explain in Chapter 7, both call and put options increase in value with volatility, so if you expect volatility to decrease, you do not want to buy a call: when volatility drops, what you have bought automatically becomes less valuable.

14. **Question:** What are the underlyings in the following derivative contracts?

(a) A life insurance contract.
(b) A home mortgage.
(c) Employee stock options.
(d) A rate lock in a home loan.

**Answer:** The underlyings are as follows:

(a) A life insurance contract: the event of one’s demise.
(b) A home mortgage: mortgage interest rate.
(c) Employee stock options: equity price of the firm.
(d) A rate lock in a home loan: mortgage interest rate.

15. **Question:** Assume you have a portfolio that contains stocks that track the market index. You now want to change this portfolio to be 20% in commodities and only 80% in the market index. How would you use derivatives to implement your strategy?

**Answer:** One would use futures to do so. We would short market index futures for 20% of the portfolio’s value, and go long 20% in commodity futures. A collection of commodity futures adding up to the 20% would be required.

16. **Question:** In the previous question, how do you implement the same trading idea without using futures contracts?

**Answer:** Futures contracts are traded on exchanges and are known as “exchange-traded” securities. An alternative approach to achieving the goal would be to use an over-the-counter or OTC product, for example, an index swap that exchanges the return on the market index for the return on a broadly defined commodity index.

17. **Question:** You buy a futures contract on the S&P 500. Is the correlation with the S&P 500 index positive or negative? If the nominal value of the contract is $100,000 and you are required to post $10,000 as margin, how much leverage do you have?

**Answer:** The futures contract is positively correlated with the stock index. The leverage is 10 times. That is, for every $1 invested in margin, you get access to $10 in exposure.
Chapter 2. Futures Markets

1. **Question**: What are “delivery options” in a futures contract? Generally, why are delivery options provided to the short but not to the long position?

   **Answer**: At settlement of a futures contract, the contract calls for the buyer to pay the seller the delivery price in exchange for the seller delivering to the buyer the specified grade of the underlying. Delivery options allow the short position to substitute an alternative grade or quality for the standard quality, at an adjustment in the delivery price. The futures contract specification lists the alternative deliverable grades and describes how the price will be adjusted for each grade.

   Delivery options are provided because specifying the deliverable grade narrowly in a commodity futures contract may limit overall supply and facilitate market corners or squeezes. Corners and squeezes are market manipulation attempts in which the manipulator takes on more long positions in a given futures contract than the short position has ability to make delivery. This is achieved by the long either controlling all of the available spot supply (a “corner”) or at least a sufficient quantity so that the short position has difficulty finding adequate deliverable supply (a “squeeze”). In a successful attempt, the price of the commodity is driven up by the lack of supply. The short position must buy the required quantity for delivery at a high price and to sell it back to the long position at the fixed price agreed to in the contract (or equivalently must compensate the long position for the difference in prices). The provision of delivery options reduces the opportunity for such behavior by the long position. For exactly the same reason, delivery options are provided only to the short position and not the long.

2. **Question**: How do delivery options affect the relationship of futures prices to forward prices?

   **Answer**: The delivery option is an option available only to the short position. The profit opportunity presented by delivery options to the short position comes at the expense of the long position. Other things being equal, the presence of delivery options means that the futures price will be lower than the forward price for a contract written on the standard grade. The presence of delivery options makes the futures contract more attractive to the short (who cannot lose from having this extra option), but less attractive to the long. With fewer buyers (long positions) and more sellers (short positions), the futures price will be lower than the forward price.

3. **Question**: To what do the following terms refer: initial margin, maintenance margin, and variation margin?

   **Answer**: An investor opening a futures account is required to deposit a specified amount of cash into an account called the margin account. The amount deposited initially is called the initial margin.

   At the end of each day, the balance in the margin account is adjusted to reflect the investor gains and losses from futures price movements over the day. This process is called marking-to-market. The changes to the margin account are called variation margin.

   A critical minimum balance amount, called the maintenance margin is specified for the margin account. If the balance in the margin account falls below this level, then the investor receives a margin call requiring the account to be topped up back to the level of the initial margin; if the top-up does not occur, the account is closed out.

4. **Question**: What are price ticks?
**Answer:** Exchanges typically place restrictions on the minimum amount by which prices may change in either direction. These are known as price ticks and vary from contract to contract. In the US, the price tick is usually around $10-$25 per contract.

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5. **Question:** Explain price limits and why they exist.

**Answer:** Exchanges often place restrictions on the maximum amount by which prices may fluctuate during a trading session. These are known as price limits. Price limits exhibit considerable variation across contracts ranging from around $1,000 per contract in some cases to over $10,000 per contract in others. In yet other cases, price limits may not exist at all, or may not be hard limits, operating, instead, in a flexible manner.

Price limits exist for the same reason as circuit breakers on the NYSE. They are aimed at preventing panics in the market, and are a function of the usual levels of volatility for the asset underlying the contract.

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6. **Question:** What are position limits in futures markets? Why do we need these? Are they effective for the objective you state, or can you think of better ways to achieve the objective?

**Answer:** To reduce the possibility of market disruption and to prevent any single trader from exercising undue influence on prices, exchanges and regulators limit the maximum number of speculative positions an investor may hold. These are called position limits. Position limits vary from contract to contract. An important determinant of the limits is the likely physical supply of the underlying commodity. Most commodity futures contracts involve position limits, whereas in markets where supply is not a constraint (e.g., Treasury or currency futures), there are often no position limits. Also, investors who qualify as bona fide hedgers do not normally face position limits, though in practice this may mean that they are allowed much higher limits than speculators.

Position limits in single contracts may not be effective in controlling overall counterparty risk, since they do not account for the other aspects of the trader’s portfolio which may contain risk-enhancing or mitigating positions, depending on the correlation of the various components of the portfolio. Other measures such as Value-at-Risk (to be studied in Chapter 20) may be more useful as they reflect the correct economic risk in contracts are usually better ways of managing total risk.

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7. **Question:** What are the different ways in which futures contracts may be settled? Explain why these exist.

**Answer:** Futures contracts may be settled (a) by delivery, (b) in cash, and (c) by exchange of physicals.

The most common is physical delivery, where the short position actually delivers the asset to the long. For contracts settled by physical delivery, delivery may be effected on any day during the delivery month.

For some financial futures contracts, especially where physical delivery is a problem (stock index futures are an example), cash settlement is used. In cash settlement, the two sides simply settle the change in contract value in cash terms. For a numerical example of how cash settlement works, consider a COMEX gold futures contract. (At the time of writing, COMEX gold futures are physically settled; we assume cash settlement only to illustrate the computations.) Suppose you enter into a long gold futures contract at a futures price of $930 per oz and the futures price at maturity is $964 per oz. Then, by buying at $930, you have gained an amount of $(964 - 930) = $34 per oz. If the contract is cash settled, you will receive $34 per oz in cash from the short futures position. Cash settlement takes place through the margin account.
Finally, a third method called exchange of physicals is also possible. This is where long and short positions with equal position sizes negotiate price and delivery terms off-exchange, and communicate the details to the exchange. There are some restrictions on how EFPs may take place; in particular, they must involve physical delivery.

8. **Question**: What is meant by open interest?

**Answer**: The open interest measures the number of futures positions that have not yet been reversed.

9. **Question**: Discuss the liquidity and maturity of futures contracts.

**Answer**: Liquidity is, in general, the ease of getting in and out of a contract. In general, most futures contracts are highly liquid at short maturities but liquidity dries up as maturity increases. One measure of liquidity for futures contracts (but not an infallible one) is the size of open interest in that contract; a high open interest indicates a large number of participants and so a relatively liquid contract.

10. **Question**: Describe the standard bond in the Treasury Bond futures contract on the CBoT and the delivery option regarding coupons.

**Answer**: The standard bond in the Treasury bond futures contract is one with a face value of $100,000, at least 15 years to maturity or first call, and a coupon of 6%.

Of the delivery options provided in the contract, the most important is the “quality option” that allows the short position to substitute any coupon for the standard 6%. The price that the long position has to pay is the quoted futures price times a conversion factor which depends on the bond that is actually delivered. The conversion factor is calculated by discounting the cash flows from the delivered bond at the standard 6% rate. If the delivered bond (a) has a coupon equal to the standard 6%, the conversion factor will be equal to one, since we are then discounting 6% cash flows at a 6% rate; (b) has a coupon higher than the standard 6%, the conversion factor will be greater than one; (c) has a coupon less than the standard 6%, the conversion factor will be less than one. See Section 2.4 and Chapter 6 for details on how the conversion factor is constructed.

11. **Question**: Suppose the delivered bond in the Treasury Bond futures contract has a remaining maturity of 20 years and a 7% coupon. Assume the last coupon was just paid. What is its conversion factor?

**Answer**: The conversion factor is

$$\frac{1}{100} \left[ \frac{3.5}{1.03} + \cdots + \frac{3.5}{1.03^{39}} + \frac{100}{1.03^{40}} \right] = 1.1156$$

12. **Question**: Suppose there are two deliverable bonds in the Treasury Bond futures contract, a 15-year 8% coupon bond and a 22-year 8% coupon bond. Assume the last coupon on both bonds was just paid. Which bond has the higher conversion factor? (Guess the answer first and then verify it by computation.)

**Answer**: Exercise for the reader. (Some exercise solutions are left to the reader intentionally. It fosters reading the text in some detail.)
13. **Question**: What is meant by the delivery grade in a commodity futures contract? What is the problem with defining the delivery grade too narrowly?

**Answer**: Since there are usually different qualities of the same commodity (e.g., different purities of gold), the delivery grade is specified in the contract to ensure that the buyer does not receive substandard quality of the commodity on delivery. But by specifying the deliverable grade too narrowly, corners and/or squeezes by the buyer are facilitated; these are situations in which the long position in the futures contract also controls all or most of the deliverable grade of the spot supply of the commodity.

14. **Question**: Identify the main institutional differences between futures contracts and forward contracts.

**Answer**: Forwards are over-the-counter (i.e., bilateral) contracts, while futures are exchange-traded. The interposition of the exchange between buyer and seller creates some differences in the contracts, as the following table summarizes:

<table>
<thead>
<tr>
<th>Futures</th>
<th>Forwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange-traded</td>
<td>Over-the-counter</td>
</tr>
<tr>
<td>Can reverse unilaterally</td>
<td>Can unwind with original counterparty, but not unilaterally reverse (e.g., by offsetting with a third party)</td>
</tr>
<tr>
<td>Default risk limited to exchange</td>
<td>Counterparty default risk exists</td>
</tr>
<tr>
<td>Margin payments required</td>
<td>No margin payments, but collateral posting may be required</td>
</tr>
<tr>
<td>Standardized contracts</td>
<td>Not standardized; fully customizable</td>
</tr>
<tr>
<td>Large numbers of participants</td>
<td>Participation generally limited to institutions and wealthy investors</td>
</tr>
</tbody>
</table>

15. **Question**: Explain the term “delivery options.” What is the rationale for providing delivery options to the short position in futures contracts? What disadvantages for hedging are created by the presence of delivery options? For valuation?

**Answer**: Delivery options are choices provided to the short position in futures markets. They allow the short position to substitute a different grade for the standardized grade. (The price to be paid by the long position upon delivery is adjusted to reflect this substitution.)

The main advantages of delivery options are that they make corners and squeezes by the long position more difficult, and, therefore, increase market integrity.

From a hedging standpoint, delivery options degrade the quality of hedge provided: the long position cannot be sure of the quality of commodity it will receive.

Delivery options complicate valuation. The futures price will depend on not just the standard grade, but on the “cheapest-to-deliver” grade (i.e., on the grade that the short position will find most profitable to deliver, given the price-adjustments for each deliverable grade and the spot prices of each grade).

16. **Question**: What is the “closing out” of a position in futures markets? Why is closing out of contracts permitted in futures markets? Why is unilateral transfer or sale of the contract typically not allowed in forward markets?
**Answer:** To close out a position in a futures market, an investor must take an offsetting opposite position in the same contract. (For example, to close out a long position in 10 S&P 500 index futures contracts with expiry in March, an investor must take a short position in 10 S&P 500 index futures contracts with expiry in March.) Once a position is closed out, the investor no longer has any obligations remaining.

*Credit risk* is key to allowing investors to close out contracts. In a futures exchange, the exchange interposes itself between buyer and seller as the guarantor of all trades; thus, there is little credit risk involved. In forward markets, allowing investors to unilaterally transfer their obligations could exacerbate credit risk, so it is typically disallowed.

An obligation under a forward contract may be eliminated in one of two ways: (a) the contract may be unwound with the same counterparty or (b) the contract may be offset by entering into an equal and opposite contract with a third party. The latter is the analog of the unilateral close-out of futures contracts. However, while close-out of the futures contract leaves the investor with no net obligations, offset of a forward contract leaves the investor with obligations on *both* contracts.

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17. **Question:** An investor enters into a long position in 10 silver futures contracts at a futures price of $4.52/oz and closes out the position at a price of $4.46/oz. If one silver futures contract is for 5,000 ounces, what are the investor’s gains or losses?

**Answer:** Effectively, the investor buys at $4.52 per oz and sells at $4.46 per oz, so takes a loss of $0.06 per oz. Per contract, this amounts to a loss of $(5000 \times 0.06) = $300$. Over 10 contracts, this results in a total loss of $3,000.00.

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18. **Question:** What is the settlement price? The opening and closing price?

**Answer:** The opening price for a futures contract is the price at which the contract is traded at the beginning of a trading session. The closing price is the last price at which the contract is traded at the close of a trading session. The settlement price is a price chosen by the exchange as a representative price from the prices at the end of a session. The settlement price is the official closing price of the exchange; it is the price used to settle gains and losses from futures trading and to invoice deliveries.

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19. **Question:** An investor enters into a short futures position in 10 contracts in gold at a futures price of $276.50 per oz. The size of one futures contract is 100 oz. The initial margin per contract is $1,500, and the maintenance margin is $1,100.

(a) What is the initial size of the margin account?

(b) Suppose the futures settlement price on the first day is $278.00 per oz. What is the new balance in the margin account? Does a margin call occur? If so, assume that the account is topped back to its original level.

(c) The futures settlement price on the second day is $281.00 per oz. What is the new balance in the margin account? Does a margin call occur? If so, assume that the account is topped back to its original level.

(d) On the third day, the investor closes out the short position at a futures price of $276.00. What is the final balance in his margin account?

(e) Ignoring interest costs, what are his total gains or losses?
**Answer:** Futures position: short 10 contracts  
Size of one contract: 100 oz  
Initial margin per contract: $1,500  
Maintenance margin per contract: $1,100  
Initial futures price: $276.50 per oz

(a) Initial size of margin account = $1,500 \times 10 = 15,000.

(b) If the settlement price is $278 per oz, the short position has effectively lost $1.50 per oz. This is a loss of $1.50 \times 100 = 150 per contract. Since the position has 10 contracts, the overall loss is $150 \times 10 = 1,500. Thus, the new balance in the margin account is $15,000 - 1,500 = 13,500. A margin call does not occur since this new balance is larger than the maintenance margin of $11,000.

(c) When the settlement price moves to $281 per oz, the short position effectively loses another $3 per oz. The loss per contract is $3 \times 100 = 300$, so the overall loss is $300 \times 10 = 3,000$. Thus, the balance in the margin account is reduced to $13,500 - 3,000 = 10,500$. Since this is less than the maintenance margin, a margin call occurs. Assume the account is topped back to $15,000.

(d) When the position is closed out at $276 per oz, the short position makes a gain of $281 - 276 = 5$ per oz. This translates to a gain of $500 per contract, and, therefore, to an overall gain of $5,000$. Thus, the closing balance in the margin account is $15,000 + 5,000 = 20,000$.

(e) The investor began with a margin account of $15,000, and deposited another $4,500 to meet the margin call, for a total outlay of $19,500. Since the margin account balance at time of close out is $20,000, his overall gain (ignoring interest costs) is $500.

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20. **Question:** The current price of gold is $642 per troy ounce. Assume that you initiate a long position in 10 COMEX gold futures contracts at this price on 7-July-2006. The initial margin is 5% of the initial price of the futures, and the maintenance margin is 3% of the initial price. Assume the following evolution of gold prices over the next five days, and compute the margin account assuming that you meet all margin calls.

<table>
<thead>
<tr>
<th>Date</th>
<th>Price per Ounce</th>
</tr>
</thead>
<tbody>
<tr>
<td>7-Jul-06</td>
<td>642</td>
</tr>
<tr>
<td>8-Jul-06</td>
<td>640</td>
</tr>
<tr>
<td>9-Jul-06</td>
<td>635</td>
</tr>
<tr>
<td>10-Jul-06</td>
<td>632</td>
</tr>
<tr>
<td>11-Jul-06</td>
<td>620</td>
</tr>
<tr>
<td>12-Jul-06</td>
<td>625</td>
</tr>
</tbody>
</table>

**Answer:** The initial margin is $321, and the maintenance margin is $193. The following is the evolution of the margin account. Note that there is one margin call that takes place on 11-July-2006.

Initiation Price = 642  
Initial Margin (5%) = 321  
Maintenance Margin (3%) = 192.6  
Number of contracts = 10
21. **Question**: When is a futures market in “backwardation”? When is it in “contango”?

**Answer**: A futures market is said to be in *backwardation* if the futures price is less than the spot price. It is in *contango* if futures price is above spot.

22. **Question**: Suppose there are three deliverable bonds in a Treasury Bond futures contract whose current cash prices (for a face value of $100,000) and conversion factors are as follows:

   (a) Bond 1: Price $98,750. Conversion factor 0.9814.
   (b) Bond 2: Price $102,575. Conversion factor 1.018.
   (c) Bond 3: Price $101,150. Conversion factor 1.004.

The futures price is $100,625. Which bond is currently the cheapest-to-deliver?

**Answer**: Since the long position will pay the futures price of 100,625 times the conversion factor in settlement, the short position prefers to deliver the bond on which the ratio of the sale price to the purchase price is highest. Essentially, this means the bond delivered is cheapest relative to the sale price. We compute this ratio for all three bonds as follows:

\[
\frac{100,625 \times 0.9814}{98,750} = 1.0000 \\
\frac{100,625 \times 1.018}{102,575} = 0.99865 \\
\frac{100,625 \times 1.004}{101,150} = 0.99879
\]

Hence, the first bond is the cheapest to deliver.

23. **Question**: You enter into a short crude oil futures contract at $43 per barrel. The initial margin is $3,375 and the maintenance margin is $2,500. One contract is for 1,000 barrels of oil. By how much do oil prices have to change before you receive a margin call?

**Answer**: If the margin account falls to a value of $2500 then a call will occur. Therefore, the loss on the position must be equal to $3375-$2500=$875 for a margin call. Solving the following equation

\[1000(P - 43) = 875\]

gives \(P = 43.875\), which is the price at which a margin call will take place.
24. **Question**: You take a long futures contract on the S&P 500 when the futures price is 1,107.40, and close it out three days later at a futures price of 1,131.75. One futures contract is for 250× the index. Ignoring interest, what are your losses/gains?

**Answer**: The gain is

\[
250(1131.75 - 1107.40) = $6087.50
\]

25. **Question**: An investor enters into 10 short futures contract on the Dow Jones Index at a futures price of 10,106. Each contract is for 10× the index. The investor closes out five contracts when the futures price is 10,201, and the remaining five when it is 10,074. Ignoring interest on the margin account, what are the investor’s net profits or losses?

**Answer**: Exercise for the reader.

26. **Question**: A bakery enters into 50 long wheat futures contracts on the CBoT at a futures price of $3.52/bushel. It closes out the contracts at maturity. The spot price at this time is $3.59/bushel. Ignoring interest, what are the bakery’s gains or losses from its futures position?

**Answer**: Each CBoT Wheat contract is for 50,000 bushels and so the settlement gain is

\[
50 \times 50,000 \times (3.59 - 3.52) = $175,000
\]

27. **Question**: An oil refining company enters into 1,000 long one-month crude oil futures contracts on NYMEX at a futures price of $43 per barrel. At maturity of the contract, the company rolls half of its position forward into new one-month futures and closes the remaining half. At this point, the spot price of oil is $44 per barrel, and the new one-month futures price is $43.50 per barrel. At maturity of this second contract, the company closes out its remaining position. Assume the spot price at this point is $46 per barrel. Ignoring interest, what are the company’s gains or losses from its futures positions?

**Answer**: Exercise for the reader.

28. **Question**: Define the following terms in the context of futures markets: market orders, limit orders, spread orders, one-cancels-the-other orders.

**Answer**: See section 2.3 of the book.

29. **Question**: Distinguish between market-if-touched orders and stop orders.

**Answer**: See section 2.3 of the book.

30. **Question**: You have a commitment to supply 10,000 oz of gold to a customer in three months’ time at some specified price and are considering hedging the price risk that you face. In each of the following scenarios, describe the kind of order (market, limit, etc.) that you would use.

(a) You are certain you wish to hedge and want to take up a futures position regardless of the price.
(b) Gold futures prices have been on an upward trend in recent days and you are not sure you want to enter the market right now. However, if the trend continues, you are afraid you will be locked into too high a price. Weighing the pros and cons, you decide you want to take a futures position if the price continues to trend up and crosses $370 per oz.

(c) Consider the same scenario as in 30(b), but now suppose also that you expect a news announcement that you think will drive gold prices sharply lower. If matters turn out as you anticipate, you want to enter into a futures position at a futures price of $350/oz or lower. However, you recognize there is a probability the news announcement may be adverse and gold prices may continue to trend up. In this case, you want to buy futures and exit if prices touch $370/oz.

(d) You want to institute a hedge only if you can obtain a gold futures price of $365/oz or less.

(e) Gold futures prices have been on a downward trend in the last few days. You are hoping this continues but don’t anticipate prices will fall too much below $362/oz, so you are willing to take the best price you can get once prices are at $364/oz.

Answer: The respective answers are:

(a) Market order.

(b) Wait till gold reaches $370 and then place a market order.

(c) Buy futures, place a sell limit order at $370 and a buy limit order at $350. If the buy limit order is hit, then close out the futures position.

(d) Place a buy limit order at $365.

(e) Place a buy limit order at $364.

31. **Question:** The spot price of oil is $75 a barrel. The volatility of oil prices is extremely high at present. You think you can take advantage of this by placing a limit order to buy futures at $70 and a limit order to sell futures at $80 per barrel. Explain when this strategy will work and when it will not.

**Answer:** This strategy has the deceptive attraction of being a buy-low and sell-high play. Of course, for it to succeed, both limits must be hit so that in effect, you will have bought low and sold high. If this does not occur then there might be losses. For example, if the buy order is hit at $70 as the oil price falls and then keeps falling, and never returns to $80 (at least before the order or the contract expire), then one eventually sustains a loss. On the other hand, if the price hits $80 and keeps on rising, then again a loss will be sustained. Thus, unless there is high volatility and a reversal of direction, this approach may not be profitable and might turn out to be loss-making.

32. **Question:** The spread between May and September wheat futures is currently $0.06 per bushel. You expect this spread to widen to at least $0.10 per bushel. How would you use a spread order to bet on your view?

**Answer:** If the price differential between September and May futures is currently $0.06 and is expected to widen to $0.10, then we should enter into a long position in the September contract and a short position in the May contract. When the spreads widens we close out both contracts.

33. **Question:** The spread between one-month and three-month crude oil futures is $3 per barrel. You expect this spread to narrow sharply. Explain how you would use a spread order given this outlook.
34. **Question**: Suppose you anticipate a need for corn in three months' time and are using corn futures to hedge the price risk that you face. How is the value of your position affected by a strengthening of the basis at maturity?

**Answer**: The basis is the futures price minus the spot price. A strengthening of the basis occurs if the basis increases. If this occurs, the position in the question is positively affected since you are long futures. In notational terms, you go long futures today (at price $F_0$, say) and close it out at $T$ (at price $F_T$, say) for a net cash flow on the futures position of $F_T - F_0$. In addition, you buy the corn you need at the time-$T$ spot price $S_T$, leading to a total net cash flow of $(F_T - F_0) - S_T = (F_T - S_T) - F_0$. A strengthening of the basis $F_T - S_T$ at maturity improves this cash flow.

35. **Question**: A short hedger is one who is short futures in order to hedge a spot cash flow risk. A long hedger is similarly one who goes long futures to hedge an existing risk. How does a weakening of the basis affect the positions of short and long hedgers?

**Answer**: The short hedger is short futures and long spot, so gains if the basis weakens. The long hedger is long futures and short spot, so loses in this case.

36. **Question**: Suppose you deliver a grade other than the cheapest-to-deliver grade on a futures contract. Would the amount you receive (the conversion factor times the futures price) exceed, equal, or fall short of the spot price of the grade you deliver?

**Answer**: By definition, the cheapest-to-deliver grade is the one for which the difference between the price received (the futures price times the conversion factor) and the spot price is greatest. Since the futures price reflects the cheapest-to-deliver grade, delivering that grade is a zero-profit situation. Consequently, delivering any other grade should lead to a loss, that is, the proceeds you receive will be less than the spot price of that grade.